



TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

SIXTY-FIRST ANNUAL MATHEMATICS CONTEST

2017

Geometry

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Scoring formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

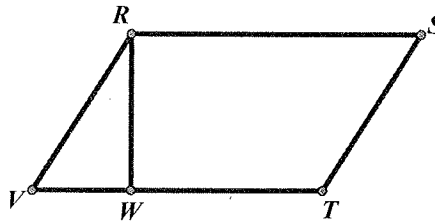
When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work

1. In trapezoid $RSTV$, $RS \parallel VT$ and $RV \perp VT$. If $RS = ST = 10$ in and $RV = 8$ in, find the area of $RSTV$.

- a. 90 in^2
- b. 112 in^2
- c. 104 in^2
- d. 120 in^2
- e. None of these

2. RW is an altitude for parallelogram $RSTV$. If $RV = 5$, $RS = 7$ and $WT = 4$, find the length of altitude RW .

- a. 3
- b. 3.5
- c. 4
- d. 4.5
- e. 5



3. Each lateral face of a regular hexagonal pyramid has lateral edges of length 10 inches and a base edge of 12 inches. What is the lateral surface area of the pyramid?

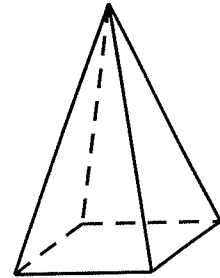
- a. 48 in^2
- b. 288 in^2
- c. 384 in^2
- d. $288 + 216\sqrt{3} \text{ in}^2$
- e. $5\sqrt{119} \text{ in}^2$

4. If the diagonals of a rhombus measure 10 cm and 24 cm, what is the perimeter of the rhombus?

- a. 13 cm
- b. 34 cm
- c. 52 cm
- d. 60 cm
- e. 68 cm

5. In regular hexagon $ABCDEF$, OM is the apothem to side AB . If $CD = 12$, find OM .
- a. 6
 - b. $6\sqrt{2}$
 - c. $6\sqrt{3}$
 - d. 8
 - e. 12

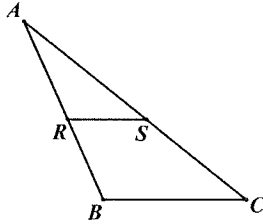
6. For the regular square pyramid shown, each base edge is 10 cm long. If the pyramid has an altitude of 12 cm, find the length l of the slant height.



- a. 5 cm
 - b. $\sqrt{62}$ cm
 - c. 12 cm
 - d. 13 cm
 - e. None of these
7. What is the number of sides in a regular polygon whose central angles each measure 24° ?
- a. 10
 - b. 12
 - c. 14
 - d. 15
 - e. 16
8. Find the exact area of a triangle whose sides measure 5, 6 and 7.
- a. $6\sqrt{5}$ units²
 - b. $6\sqrt{6}$ units²
 - c. $6\sqrt{7}$ units²
 - d. 15 units²
 - e. None of these

9. For triangle ABC , $RS \parallel BC$. If $AR = 5$, $RB = 2.5$ and $RS = 4$, find BC .

- a. 2
- b. 3
- c. 4
- d. 6
- e. 7.5

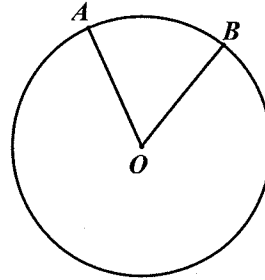


10. Which of the following lines (line segments) are concurrent?

- a. Angle-bisectors of a rectangle
- b. Perpendicular-bisectors of the sides of a rhombus
- c. Perpendicular-bisectors of the sides of a kite
- d. Angle-bisectors of a trapezoid
- e. Angle-bisectors of a kite

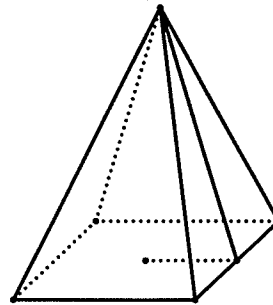
11. In circle O , $m\widehat{AB} = 45^\circ$ and $OA = 6$ cm. Find $\ell\widehat{AB}$.

- a. 1.5π cm
- b. 3π cm
- c. 6π cm
- d. 12π cm
- e. 270 cm



12. The sides of the square base of a regular pyramid measure 8 inches each while the altitude measures 10 inches. To the nearest degree, find the measure of the angle formed by the slant height and the apothem of the square base of the pyramid shown.

- a. 56°
- b. 60°
- c. 64°
- d. 66°
- e. 68°



13. The volume of a right circular cone is exactly $81\pi \text{ ft}^3$. If the altitude of the cone has a length of 12 feet, what is the length of radius of the circular base?

- a. 4 ft
- b. 4.25 ft
- c. 4.5 ft
- d. 4.75 ft
- e. 5 ft

14. The measures of two supplementary angles are in the ratio 2:3. Find the measure of the larger angle.

- a. 36°
- b. 54°
- c. 72°
- d. 108°
- e. 120°

15. For the right triangle with sides of lengths 5 in, 12 in and 13 in, find the length of the radius of the inscribed circle.

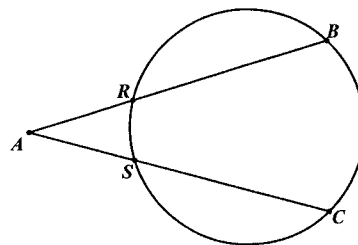
- a. 4 in
- b. 3.5 in
- c. 3 in
- d. 2.5 in
- e. 2 in

16. For a triangle RST , points M , N and P are the midpoints of the sides. If the perimeter of $\triangle RST$ is 47 inches, find the perimeter of $\triangle MNP$.

- a. 11.75 in
- b. 23 in
- c. 23.5 in
- d. 47 in
- e. None of these

17. If $m\widehat{BC} = 105^\circ$ and $m\widehat{RS} = 27^\circ$, find $m\angle BAC$.

- a. 88°
- b. 78°
- c. 66°
- d. 44°
- e. 39°



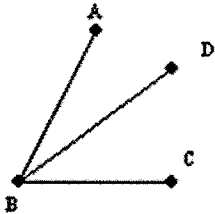
18. In a triangle RST , $\overline{RS} \cong \overline{ST}$. Which statement is not necessarily true?

- a. $\angle R \cong \angle T$
- b. $\angle S$ is the vertex angle of $\triangle RST$
- c. RT is the base of $\triangle RST$
- d. $RS + ST > RT$
- e. RT is the longest side of $\triangle RST$

19. For trapezoid $RSTV$, $\overline{RS} \parallel \overline{VT}$ and \overline{MN} is the median. If $RS = 6.5$, $ST = 8.5$, $VT = 14.7$ and $VR = 9.7$, then the length of MN is

- a. 10.6
- b. 15.0
- c. 9.85
- d. 9.1
- e. 21.2

20. If $m\angle ABD = 2x + 17$, $m\angle DBC = 4(x - 3)$, and $m\angle ABC = 8x - 12$, find x .



- a. $x = 7.75$
- b. $x = 8$
- c. $x = 8.25$
- d. $x = 8.5$
- e. $x = 8.75$

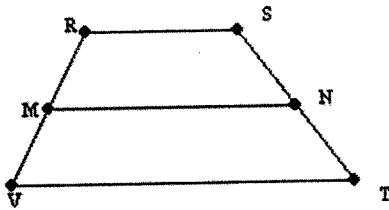
21. In space, determine the locus of points equidistant from the endpoints of \overline{AB} .

- a. A line
- b. A circle
- c. A sphere
- d. A plane
- e. An ellipsoid

22. The measures of the interior angles of $\triangle ABC$ are in the ratio 2:3:7. What type of triangle is $\triangle ABC$?

- a. Acute
- b. Right
- c. Obtuse
- d. Isosceles
- e. Equilateral

23. For trapezoid $RSTV$, $\overline{RS} \parallel \overline{VT}$. If M is the midpoint of \overline{RV} and N is the midpoint of \overline{ST} , it follows that:



- a. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$ and $MN = RS + VT$
- b. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$ and $MN = \frac{1}{2}(RV + ST)$
- c. $\overline{MN} \parallel \overline{VT}$ and $MN = RS + VT$
- d. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$ and $MN = \frac{1}{2}(RS + VT)$
- e. $\overline{MN} \parallel \overline{RS}$, $\overline{MN} \parallel \overline{VT}$, MN cannot be determined

24. In the form $y = mx + b$, find the equation of the line that contains (g, h) and is perpendicular to $y = cx + d$.

- a. $y = -\frac{1}{c}x + [\frac{g}{c} + h]$
- b. $y = \frac{1}{c}x + [\frac{g}{c} + h]$
- c. $y = cx + [\frac{g}{c} + h]$
- d. $y = -cx + h$
- e. None of these

25. Determine the exact volume of the sphere $(x - 2)^2 + (y + 3)^2 + (z - 4)^2 = 36$.

- a. 336π units³
- b. 288π units³
- c. 168π units³
- d. 148π units³
- e. 118π units³

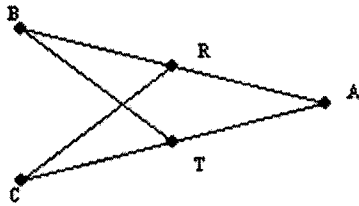
26. For the triangle whose perimeter measures 36 units, the radius of the inscribed circle is 3. Find the area of the triangle.

- a. 18 units²
- b. 27 units²
- c. 54 units²
- d. 108 units²
- e. 48 units²

27. A right circular cone has a diameter for its base of length 16 inches and an altitude of 15 inches. Find the exact lateral surface area of the cone.

- a. 120π in²
- b. 240π in²
- c. 136π in²
- d. 156π in²
- e. $\sqrt{481}\pi$ in²

28. In the figure, $\angle B \cong \angle C$. If $m\angle ARC = x + 50$ and $m\angle BTC = x$, find the value of x .

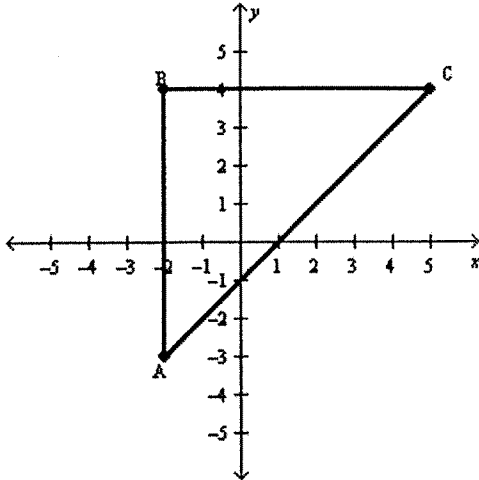


- a. 55°
- b. 65°
- c. 75°
- d. 85°
- e. 95°

29. In isosceles trapezoid, $JK \parallel HL$. If $m\angle H = 45^\circ$, $JK = 6$ cm and the length of an altitude is 4 cm, find the exact perimeter of $HJKL$.

- a. 24 cm
- b. $(20 + 4\sqrt{2})$ cm
- c. $(20 + 4\sqrt{3})$ cm
- d. $(20 + 8\sqrt{3})$ cm
- e. $(20 + 8\sqrt{2})$ cm

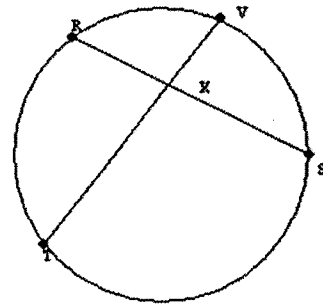
30. Find the area of $\triangle ABC$, which has vertices at $A(-2, -3)$, $B(-2, 4)$ and $C(5, 4)$.



- a. $\frac{49}{3}$ units²
- b. $\frac{49}{2}$ units²
- c. 49 units²
- d. 98 units²
- e. $14 + 7\sqrt{2}$ units²

31. In the figure, chords RS and TV intersect at point X . If $RX = 8$, $TX = 12$ and $XS = 9$, find XV .

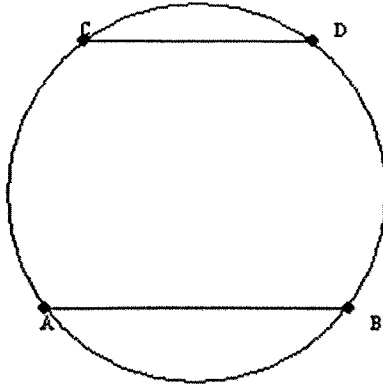
- a. 5
- b. 5.6
- c. 6
- d. 7.5
- e. 7



32. In circle O , radius $\overline{OD} \perp$ chord \overline{AB} at point C . If $OD = 5$ and $AB = 8$, find OC .

- a. 2.5
- b. 3
- c. 3.5
- d. 4
- e. $\sqrt{39}$

33. The length of radius of the circle shown is 10 inches. If $CD = 12$ inches and $AB = 16$ inches, how much closer to the center is chord AB than CD ?



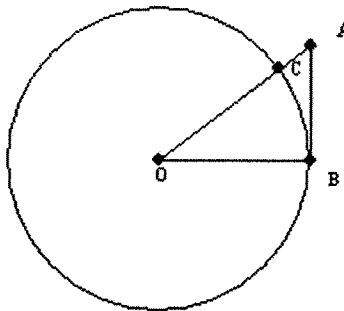
- a. 2 inches
b. 4 inches
c. 6 inches
d. 8 inches
e. 14 inches
34. When parallel lines a and b are cut by transversal d , $\angle 2$ and $\angle 7$ are a pair of alternate exterior angles that are formed. If $m\angle 2 = x^2 + 7x$ and $m\angle 7 = x(x - 3) + 40$, find x .
- a. $x = 3$
b. $x = 3.5$
c. $x = 4$
d. $x = 4.5$
e. $x = 5$
35. In an arbitrary isosceles triangle, which of the following must be true?
- a. The three medians are congruent.
b. The medians to the congruent sides are equal in length.
c. The medians to the congruent sides are perpendicular.
d. The median to the base is longer than the others medians.
e. None of these.

36. In circle P , $m\widehat{AC} = 60^\circ$ and $BC = 12$. Find AB if BC is a diameter.

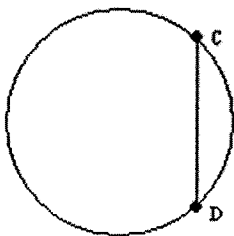
- a. 3
- b. $3\sqrt{3}$
- c. $4\sqrt{3}$
- d. 6
- e. $6\sqrt{3}$

37. AB is tangent to circle O at point B . If $OB = 4$ and $AB = 3$, find AC .

- a. 1
- b. 1.2
- c. 1.5
- d. 2
- e. 2.2



38. For the segment bounded by chord \overline{CD} and arc \widehat{CD} , $CD = 12\sqrt{3}$ and $m\widehat{CD} = 120^\circ$. Find the exact area of the segment.



- a. $48\pi \text{ cm}^2$
- b. $(48\pi - 18\sqrt{3}) \text{ cm}^2$
- c. $(48\pi + 18\sqrt{3}) \text{ cm}^2$
- d. $(48\pi - 18\sqrt{2}) \text{ cm}^2$
- e. $(48\pi - 36\sqrt{3}) \text{ cm}^2$

39. For a quadrilateral to be a rectangle, how must the diagonals be related?

- a. Parallel
- b. Perpendicular
- c. Congruent
- d. Bisect each other
- e. Congruent and bisect each other

40. Let the vertices of a parallelogram $MNPQ$ be the points $M(0,0)$, $N(a,0)$, $P(a+b,c)$, and $Q(b,c)$. Which equation must hold true if the diagonals are perpendicular?

a. $\frac{c}{a+b} = \frac{c}{b-a}$

b. $\frac{c}{a+b} \cdot \frac{c}{b-a} = 1$

c. $\frac{c}{a+b} \cdot \frac{c}{b-a} = -1$

d. $\frac{c}{a+b} = -\frac{c}{b-a}$

e. None of the above