Calculus & Advanced Topics Spring 2011

1. The derivative of $x^2 \cos(x)$ is

A. $-2x \sin(x)$ B. $-x^2 \sin(x)$ C. $2x \cos(x) - x^2 \sin(x)$ D. $2x - \sin(x)$ E. $2x \cos(x)$

2. $\int_{-1}^{3}(3-x)dx =$

A. 1 B. 2 C. 4.5 D. 7 E. 8

3. $\lim_{x\to 3} \frac{x^2-9}{x^2-x-6} =$

A. 0 B. 1 C. 6/5 D. 3/2 E. undefined

4. Find the derivative of $\cos^2(x^2-5)$

A. $2 \cos(x^2-5)$
B. $-2 \sin(2x)$
C. $-4x \sin(x^2-5)$
D. $-4x \cos(x^2-5) \sin(x^2-5)$
E. $-4x \cos(x^2-5) + 2x \sin(x^2-5)$

5. How many solutions are there to the equation $x^2 = 2^x$?

A. 0 B. 1 C. 2 D. 3 E. 4

6. If $0 \le x \le 1$, then $\sin(\arccos(x))$ =

A. $\sqrt{1-x^2}$ B. $1 + x^2$ C. $\arcsin(x)$ D . $\sin(x^2+1)$ E. impossible to determine

7. $\int \left(\frac{1}{x}+\frac{1}{x^2}\right) dx =$

A. $\frac{2}{x^2}+\frac{3}{x^3}+C$
B. $\ln(|x|)+\frac{3}{x^3}+C$
C. $\ln(|x|)-\frac{3}{x^3}+C$
D. $\frac{2}{x^2}-\frac{1}{x}+C$
E. $\ln(|x|)-\frac{1}{x}+C$

8. The derivative of arctan(2x+3) [that is, the inverse tangent of 2x+3] is

A. 2 $\text{arcsec}^2(2x+3)$ B. $\frac{2}{4x^2+12x+10}$ C. $\frac{1}{1+(2x+3)^2}$ D. $\frac{2}{\sqrt{1-(2x+3)^2}}$ E. $\frac{2}{1+x^2}$

9. Which of the following curves has a vertical asymptote at x =3 and a horizontal asymptote at y = 2?

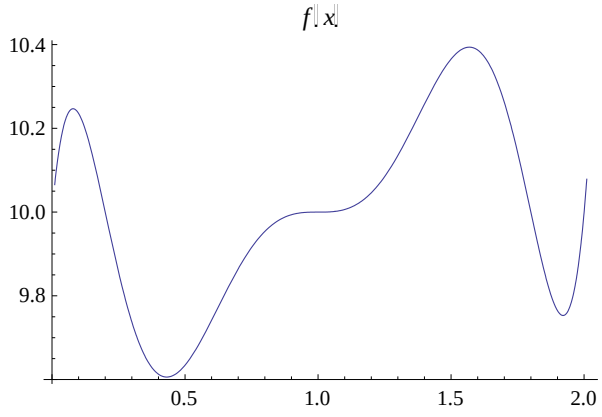
A. $y=\frac{2x^2-1}{x^2-4x+3}$ B. $y=\frac{2x-1}{x^2-9}$ C. $y=\frac{x-2}{x-3}$ D. $y=\frac{2x^2}{x^2+6x+9}$ E. $y=\frac{x+2}{x^2+3}$

10. Compute the derivative of the function $f(x)=\frac{e^{3x}-x}{x+1}$.

A. $\frac{e^{3x}(3x+2)-x}{(x+1)^2}$ B. $\frac{e^{3x}(3x+2)+x}{(x+1)^2}$ C. $\frac{3e^{3x}-1}{(x+1)^2}$ D. $\frac{e^{3x}(3x+2)-1}{(x+1)^2}$ E. $\frac{e^{3x}-x}{(x+1)^2}$

11. The function $f(x)$ graphed below has how many critical points in the open interval (0, 2)?

A. 1
B. 4
C. 5
D. 6
E. 7



12. Compute $\lim_{x\to 0} \frac{2^{3x}-1}{3x}$.

A. ∞ B. ln 2 C. ln 3 D. ½ E. 1

13. "Throughout the spring, the amount of daylight each day increases, but it increases more and more slowly." Suppose *G(t)* is a continuous function with first and second derivatives that predicts the amount of daylight on day *t* ; that is, *G(t)* ≈ the number of hours of daylight *t* days after the start of spring. What does the sentence in quotes tell you about *G'(t)* and *G''(t)* ?

A. *G'(t)* > 0 and *G''(t)* > 0
B. *G'(t)* > 0 and *G''(t)* < 0
C. *G'(t)* < 0 and *G''(t)* > 0
D. *G'(t)* < 0, and the sentence tells you nothing about *G''(t)*
E. The sentence tells you nothing about either *G'(t)* or *G''(t)*

14. Suppose $F'(x) = \sin(mx)$, where *m* is a positive real number, and $F(0) = \frac{2}{m}$.
An expression for *F(x)* is

A. $\frac{1}{m}\sin(mx) - \frac{2}{m}$
B. $m\sin(mx) - \frac{2}{m}$
C. $-\frac{1}{m}\cos(mx) + \frac{1}{m}$
D. $-\frac{1}{m}\cos(mx) + \frac{3}{m}$
E. $\frac{1}{m}\sin(mx) + \frac{3}{m}$

15. If $xy + \cos(y) = x^2$, then $\frac{dy}{dx}$ is

A. $\frac{2x-y}{x-\sin(y)}$ B. $\frac{2x-y+\sin(y)}{x}$ C. $\frac{2x+\sin(y)}{x}$ D. $2x - y + \sin(y)$ E. $\frac{x^2+x\sin(y)+\cos(y)}{x^2}$

16. $\lim_{x\to 2} \frac{2-x}{1-\sqrt{x-1}}$ =

A. 0 B. 1 C. $\sqrt{2}$ D. 2 E. does not exist

17. $\int x\cos(2x)\,dx =$

A. $x^2\sin(2x) + C$
B. $\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C$
C. $\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + C$
D. $\frac{1}{2}x\sin(2x) + C$
E. $\frac{1}{4}x^2\sin(2x) + C$

18. Find the solution to the equation $e^{2x} - 2^x = x^2 - 2$ on the interval $[-5,5]$.

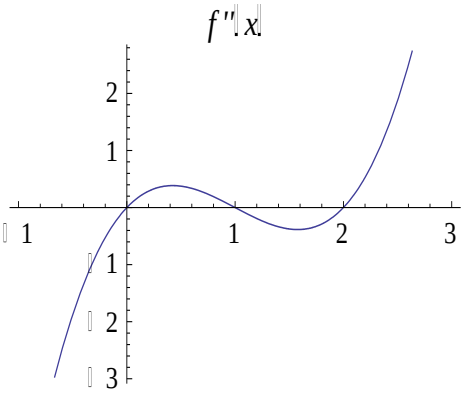
A. $x \approx -0.822992$
B. $x \approx -1.291112$
C. $x \approx -0.145451$
D. $x \approx -1.545343$
E. $x \approx -1.114554$

19. Find the equation of the line tangent to the graph of $f(x) = x^3 - 4x^2 + 2$ at the point where $x = 1$.

A. $y = -5x - 1$
B. $y = -5x + 4$
C. $y = -3x - 1$
D. $y = (6x^2 - 4x + 1)x + 4$
E. $y = (6x^2 - 4x + 1)x - 1$

20. Given the graph of $f''(x)$, for what values of x in (-1, 3) is $f(x)$ concave up?

A. (0,1)
B. (1,2)
C. $(-1,0) \cup (1,2)$
D. $(-1,0) \cup (1,2)$
E. $(0,1) \cup (2, 3)$



21. Find the *x*-values of all critical points of the function $f(x) = x^2 e^{-x^2}$.

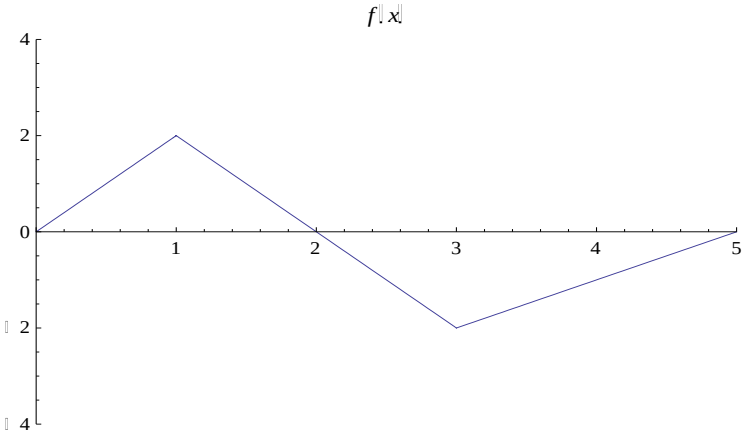
A. $x = -1, 1$
B. $x = -1, 0, 1$
C. $x = -\ln 2, 0, \ln 2$
D. $x = -\ln 2, \ln 2$
E. $x = 0$

22. Use the fact that $\int_3^5 (4f(x) + 3)dx = 26$ to compute $\int_3^5 f(x)dx$.

A. 6.5 B. 5.75 C. 3 D. 5 E. 2.5

23. Use the plot of $f(x)$ below to compute $\int_1^5 f(x)\, dx$.

A. –2
B. –1
C. 0
D. 1
E. 3



24. Minerva lives 3 miles directly east of town. She drives to town and then turns onto the highway heading north, traveling at 50 miles per hour. When she is 5 miles away from town, at what rate is her distance from her home increasing?

A. $\frac{250}{\sqrt{34}} \approx 42.9$ mph B. 50 mph C. 30 mph D. 34 mph E. $10\sqrt{34} \approx 58.3$ mph

25. A ball is thrown upward from the top of a 64 ft. building with an initial velocity of 48 feet per second. (Note: The acceleration due to gravity is 32 ft./sec^2. Ignore air resistance.) How fast is the ball going when it hits the ground?

A. 32 ft./sec B. 48 ft./sec. C. 60 ft./sec. D. 64 ft./sec. E. 80 ft./sec.

26. Which of the following series converges?

A. $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ B. $\sum_{n=1}^{\infty} \left(\frac{3}{\pi}\right)^n$ C. $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ D. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ E. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

27. Find the interval over which $f(x) = \int_0^x \frac{1}{1+t+t^2}\,dt$ is concave up.

A. $\frac{1}{2} < x < \infty$
B. $-\infty < x < \frac{1}{2}$
C. $-\frac{1}{2} < x < \infty$
D. $-\infty < x < -\frac{1}{2}$
E. $-\infty < x < 1$

28. If $y = x^{\sin(x)}$, then $\frac{dy}{dx}$ is

A. $\sin(x)\, x^{\sin(x)-1}$
B. $x^{\sin(x)}\cos(x)$
C. $x^{\sin(x)}\cos(x)\ln(x)$
D. $\sin(x)\, x^{\sin(x)-1} + x^{\sin(x)}\cos(x)\ln(x)$
E. $\cos(x)\ln(x) + \frac{\sin(x)}{x}$

29. A T-shirt vendor sells 30 T-shirts a day for $10 each. She estimates that for each $1 she raises the price, she will sell 4 fewer shirts a day, or for each $1 she lowers the price, she will sell 4 more shirts a day. What price that will bring her the maximum revenue?

A. $8.75 B. $9.50 C. $10.00 D. $10.75 E. $12.00

30. An unfair coin flips heads with probability 2/3. You begin flipping the coin. What is the probability that the first tails you see is on the 8th coin flip?

A. 256/6561 B. 128/6561 C. 1/6561 D. 1120/6561 E. 2/6561

31. The graphs of the equations $y = 4x - x^3$ and $y = x + 2$ are tangent to each other at the point

A. (-2,0) B. (-1,-3) C. (-1,1) D. (0,0) E. (1,3)

32. Compute $\lim_{x\to\infty} \frac{4\cos(e^{3x}+x^4)}{x}$.

A. ∞ B. 1 C. 0 D. –1 E. $-\infty$

33. Observe that $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ is a decreasing function over the interval [0, $+\infty$). Using a Riemann sum with four subintervals and sampling the function at the left-hand sides of the subintervals, estimate $\frac{1}{\sqrt{2\pi}} \int_0^1 e^{-\frac{x^2}{2}}\, dx$ and state whether your estimate is an overestimate, an underestimate, or exactly equal to the integral.

A. ≈ 0.359703, and this answer is an underestimate
B. ≈ 0.359703, and this answer is an overestimate
C. ≈ 0.359703, and this answer is exactly equal to the integral we are approximating
D. ≈ 0.341345, and this answer is an underestimate
E. ≈ 0.341345, and this answer is an overestimate

34. $\int_{-\infty}^{0} e^{2x-1}\, dx =$

A. $\frac{1}{2e}$ B. $\frac{1}{2}$ C. $\frac{e}{2}$ D. 2 E. $+\infty$

35. Consider the series $S_\infty = \sum_{n=1}^{\infty} (-1)^n \frac{n+3}{3n^2-n+1}$ and define the N^{th} partial sum S_N by $S_N = \sum_{n=1}^{N} (-1)^n \frac{n+3}{3n^2-n+1}$. The error of the N^{th} partial sum is $|S_\infty - S_N|$. What is the smallest upper bound on the error S_{40} that you can justify?

A. The error is bounded from above by 88/5003.
B. The error is bounded from above by 44/5003.
C. The error is bounded from above by 50/1000.
D. The error is bounded from above by 60/1000.
E. The series has converged by S_{40}. There is no longer any error.

36. Air is being pumped into a spherical balloon at a rate of 5 cm^3/min. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 100 cm.

A. $1/(1000\pi)$ centimeters per minute
B. $3/(4\pi)$ centimeters per minute
C. $1/(4000\pi)$ centimeters per minute
D. $4/\pi$ centimeters per minute
E. $1/(2000\pi)$ centimeters per minute

37. Suppose $g(x)$ is a continuous function for all real x, $g(0) = 5$ and $g(10) = 12$. Which of the following equations must have a solution in the interval $(0, 10)$?

A. $g(x) = 5$ B. $g(x) = 12$ C. $g(x) = 13$ D. $g(x) = 6$ E. $g(x) = 0$

38. Which of the following functions satisfies $x\frac{dy}{dx} = 3y - \frac{5}{2}\sqrt{x} - 12$?

A. $y = x^3$
B. $y = x^3 - \sqrt{x} + 4$
C. $y = x^3 + \sqrt{x} + 4$
D. $y = x^3 + \sqrt{x}$
E. $y = x^3 - \sqrt{x}$

39. For which of the following values of *a* and *b* is $\int_a^b \sin^3 x \ dx = 0$?

A. $a = 0, \ b = \pi$
B. $a = 0, \ b = 3\pi$
C. $a = 2\pi, \ b = 3\pi$
D. $a = \frac{\pi}{2}, \ b = \frac{3\pi}{2}$
E. $a = \frac{\pi}{2}, \ b = \pi$

40. $\lim_{h\to 0} \frac{e^{2(x+h)} - e^{2x}}{h} =$

A. $2e^{2x}$ B. e^{2x} C. 1 D. 2 E. limit does not exist