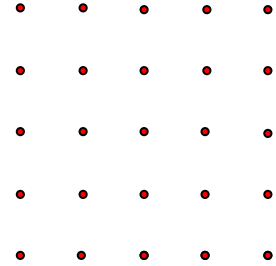


Geometry Contest for 2010

- 1) Two angles are complementary if their sum is  
A)  $180^\circ$     B) between  $90^\circ$  and  $180^\circ$     C)  $90^\circ$     D) less than  $90^\circ$     E) more than  $180^\circ$
  
- 2) If two exterior angles of a triangle are congruent, then the triangle is  
A) acute    B) scalene    C) isosceles    D) obtuse    E) equilateral
  
- 3) Assume that quadrilateral ABCD is a parallelogram. To completely prove "ABCD is a rectangle if and only if  $AC = BD$ " we need to prove:  
A) If ABCD is a rectangle, then  $AC = BD$ .  
B) If  $AC = BD$  in parallelogram ABCD, then ABCD is a rectangle.  
C) If ABCD is not a rectangle, then  $AC \neq BD$ .  
D) If ABCD is a rectangle with  $AC = BD$ , then ABCD is a parallelogram.  
E) Need to prove both (A) and (B).
  
- 4) If two parallel lines are cut by a transversal, then the  
A) Interior angles on the same side of the transversal are congruent.  
B) Alternate interior angles are supplementary.  
C) Alternate interior angles are congruent.  
D) Corresponding angles are supplementary.  
E) Alternate exterior angles are supplementary.
  
- 5) Which of these is the contrapositive of "If I have a nickel, then I am rich?"  
A) If I am rich, then I have a nickel.  
B) If I don't have a nickel, then I am not rich.  
C) If I am not rich, then I don't have a nickel.  
D) If I have a nickel, then I am rich.  
E) I am rich if and only if I have a nickel
  
- 6) If two distinct planes intersect, then their intersection is  
A) two parallel lines    B) a line    C) a point    D) a plane    E) a plane crash

- 7) How many different length line segments are there whose endpoints are on a 5 X 5 geoboard?  
 A) 4    B) 5    C) 8    D) 9    E) 14

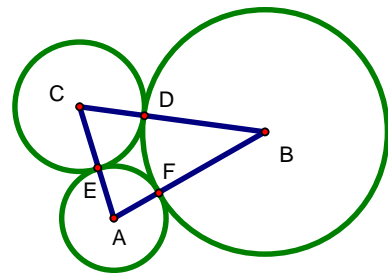


- 8) If the sides of a triangle are 20, 21, and 29 units, then the triangle is  
 A) acute    B) isosceles    C) obtuse    D) right    E) not possible

- 9) The circles with centers at A, B, and C are mutually tangent at D, E, and F as shown. Compute.

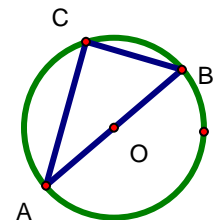
$$\frac{AF \cdot BD \cdot CE}{FB \cdot DC \cdot EA}$$

- A)  $\frac{1}{2}$     B) 1    C)  $\emptyset$     D)  $2\pi$     E) cannot be determined

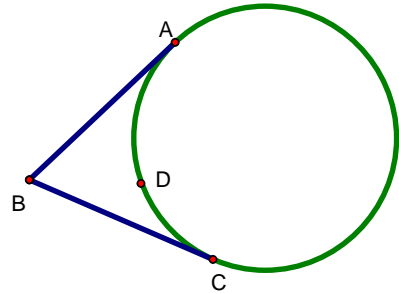


- 10)  $\triangle ABC$  is inscribed in a circle with diameter  $\overline{AB}$ . If  $m \widehat{BC} = 30^\circ$  (the measure of arc BC) =  $30^\circ$ , find the measure of  $\angle ABC$ .

- A)  $60^\circ$     B)  $150^\circ$     C)  $75^\circ$     D)  $90^\circ$     E)  $15^\circ$

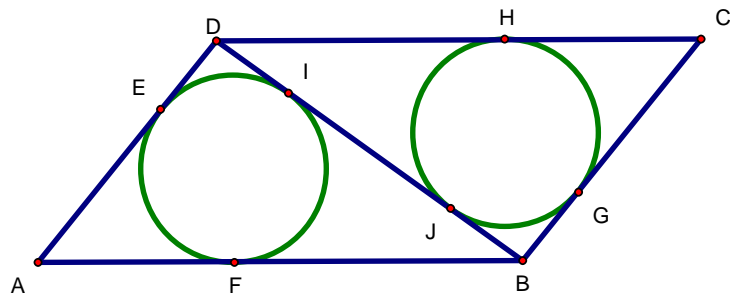


- 11) If A and C are points of tangency to the circle and D is an arbitrary point on minor arc  $\widehat{AC}$ , find  $m\angle B + m\widehat{ADC}$ .  
 A)  $90^\circ$     B) between  $90^\circ$  and  $180^\circ$     C)  $180^\circ$     D) between  $180^\circ$  and  $360^\circ$     E)  $360^\circ$



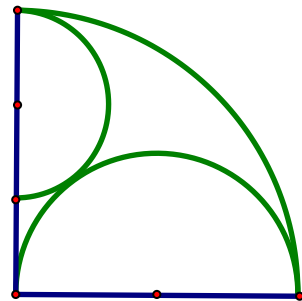
- 12) Segment  $\overline{BD}$  is a diagonal in parallelogram ABCD. Incircles of  $\triangle ABD$  and  $\triangle BCD$  have points of tangency at E, F, G, H, I, and J as shown. Find IJ:

- A)  $\frac{AB+BC}{3}$     B)  $\sqrt{AB \cdot BC}$     C)  $AB - BC$     D)  $\frac{AB+BC}{3}$     E)  $AB + BC - DI - JB$

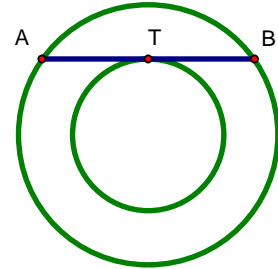


- 13) Two semicircles are constructed in a quadrant of a circle as shown. If the diameter of the larger semicircle and the radius of the quadrant are each 8 units, find the radius of the smaller semicircle.

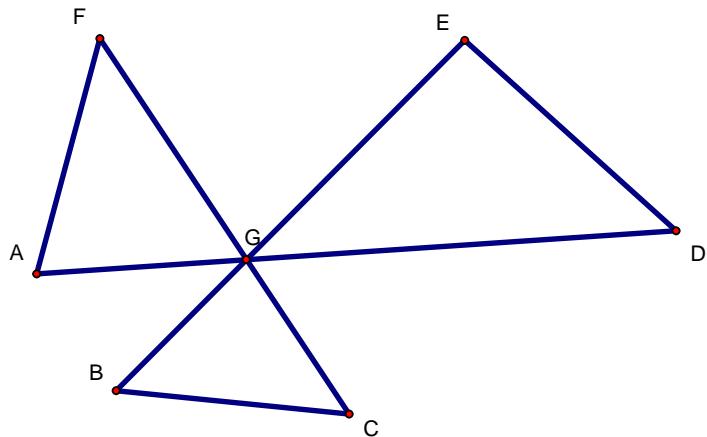
- A)  $\frac{2}{3}$     B) 1    C) 2    D)  $\frac{8}{3}$     E) 4



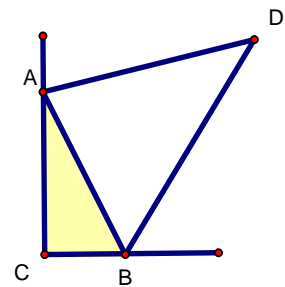
- 14) Find the area of the “ring” between two concentric circles if chord  $\overline{AB}$  of the larger circle is tangent at point T of the smaller circle and  $AB = 8$ .
- A)  $2\pi$       B)  $8\pi$       C)  $12\pi$       D)  $16\pi$       E) insufficient information to solve.



- 15) The three triangles in the figure are scalene. Segments  $\overline{AD}$ ,  $\overline{BE}$ , and  $\overline{CF}$  are all concurrent at G. Find  $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F$ .
- A)  $90^\circ$       B)  $180^\circ$       C)  $270^\circ$       D)  $360^\circ$       E) cannot be determined



- 16) Rays  $\overrightarrow{AD}$  and  $\overrightarrow{BD}$  bisect exterior angles of  $\triangle ABC$ . If  $m\angle C = 90^\circ$ , find  $m\angle D$ .
- A)  $30^\circ$       B)  $45^\circ$       C)  $60^\circ$       D)  $75^\circ$       E) cannot be determined



17) Given:  $\triangle ABC \sim \triangle BED$  with  $DB = 4$ ,  $BE = 6$ ,  $EC = 2$ , and  $ED = 4$ . Find the area of  $\triangle ABC$ .

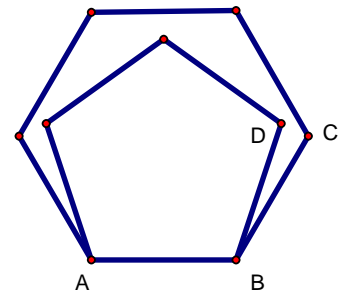
- A) 14      B) 28      C) 56      D) 63      E)  $12\sqrt{7}$

18) Four triangles have sides of lengths (in cm) as given by: a) 5 – 5 – 6 b) 5 – 5 – 7 c) 5 – 5 – 8 d) 5 – 6 – 7 Which one has the largest area?

- A) 5-5-6    B) 5-5-7    C) 5-5-8    D) 5-6-7    E) All four areas have the same measure.

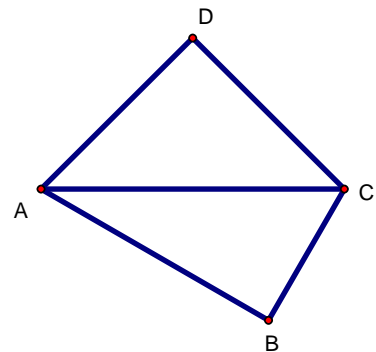
19) A regular pentagon is inside a regular hexagon and shares side  $\overline{AB}$  as shown. Find  $m\angle DBC$ .

- A)  $12^\circ$     B)  $20^\circ$     C)  $36^\circ$     D)  $60^\circ$     E)  $8\frac{1}{2}^\circ$

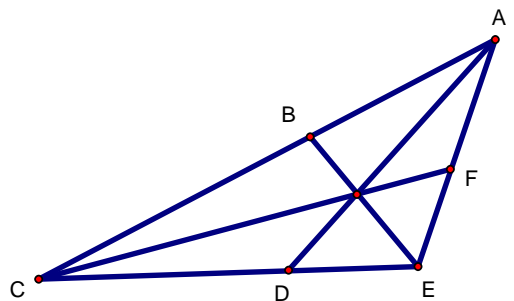


20) If  $\triangle ABC$  is a  $30^\circ - 60^\circ - 90^\circ$  triangle and  $\triangle ADC$  is a  $45^\circ - 45^\circ - 90^\circ$  triangle, find the ratio of the area of  $\triangle ABC$  to the area of  $\triangle ADC$ .

- A) 1      B)  $\sqrt{2}$     C)  $\sqrt{3}$     D)  $\sqrt{\frac{3}{2}}$     E)  $\frac{\sqrt{3}}{2}$



- 21) Which of the following does not form a regular tessellation ?
- A) an equilateral triangle
  - B) a regular polygon of 4 sides
  - C) a regular pentagon
  - D) a regular hexagon
  - E) All of the above form regular tessellations.
- 22) Let ABCD be a general convex quadrilateral whose diagonals meet at E. Let F, G, H and I be the centroids of  $\triangle AEB$ ,  $\triangle BEC$ ,  $\triangle CED$ , and  $\triangle DEA$ , respectively. What kind of quadrilateral must FGHI be?
- A) Square
  - B) Rhombus
  - C) Rectangle
  - D) Parallelogram
  - E) Trapezoid
- 23) How many diagonals are there in a convex hexagon?
- A) 3
  - B) 5
  - C) 6
  - D) 8
  - E) 9
- 24) If each of the dimensions of a cube is doubled to form a new cube, then what is the ratio of the volume of the original cube to the volume of the new cube?
- A)  $\frac{2}{1}$
  - B)  $\frac{1}{2}$
  - C)  $\frac{1}{4}$
  - D)  $\frac{8}{1}$
  - E)  $\frac{1}{8}$
- 25) Given an arbitrary triangle which of the following concurrency points are always collinear?
- A) orthocenter, incenter , centroid
  - B) circumcenter, incenter, centroid
  - C) circumcenter, incenter, centroid
  - D) orthocenter, centroid, circumcenter
  - E) centroid, incenter, orthocenter
- 26) If  $\overline{AD}$ ,  $\overline{BE}$  and  $\overline{CF}$  are concurrent, with  $AB = 6$ ,  $BC = 8$ ,  $CD = 4$ ,  $DE = 3$ ,  $EF = 2$ , and  $FA = x$ , then the value of x is
- A) 1
  - B) 2
  - C) 3
  - D) 4
  - E) 5



27) If two angles of a triangle are  $17^\circ$  and  $43^\circ$ , find the measure of the largest exterior angle.

- A)  $60^\circ$     B)  $120^\circ$     C)  $163^\circ$     D)  $137^\circ$     E)  $300^\circ$

28) How many diagonals are in a convex polygon with nine sides?

- A) 9    B) 8    C) 7    D) 16    E) 27

29) Find the number of sides of a regular polygon if each exterior angle is  $9^\circ$ .

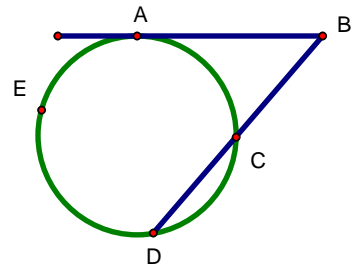
- A) 9    B) 40    C) 36    D) 27    E) 171

30) Chords  $\overline{AB}$  and  $\overline{CD}$  intersect at a point E inside a circle. If  $AE = 12$ ,  $BE = 3$ , and E is the midpoint of  $\overline{CD}$ , then find the length of  $\overline{CD}$ .

- A) 15    B) 36    C) 6    D) 12    E) 9

31) In the figure,  $\overline{AB}$  is tangent to the circle at A and  $\overline{BD}$  intersects the circle again at C. If E is a point on arc  $\widehat{AD}$  remote from B,  $m\widehat{AED} = 210^\circ$ , and  $m\angle B = 65^\circ$ , find  $m\widehat{AC}$ .

- A)  $65^\circ$     B)  $145^\circ$     C)  $130^\circ$     D)  $80^\circ$     E)  $137.5^\circ$

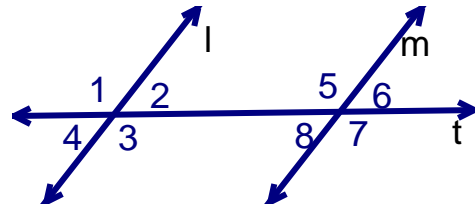


32) If M is the midpoint of  $\overline{AB}$ ,  $AM = 2x+3$ , and  $MB = 3(x-2)$ , find AB.

- A) 9    B) 18    C) 21    D) 42    E) 30

33) In the figure  $t$  is a transversal for parallel lines  $l$  and  $m$ . If  $m \angle 2 = x + y$ ,  $m \angle 5 = 3x + y$ , and  $x - y = 15^\circ$ , find  $m \angle 4$ .

- A)  $15^\circ$     B)  $20^\circ$     C)  $35^\circ$     D)  $45^\circ$     E)  $55^\circ$



34) In a regular tetrahedron planes parallel to each face pass through the midpoints of the remaining edges. If all such planes are considered at once, then how many smaller regular tetrahedra are formed?

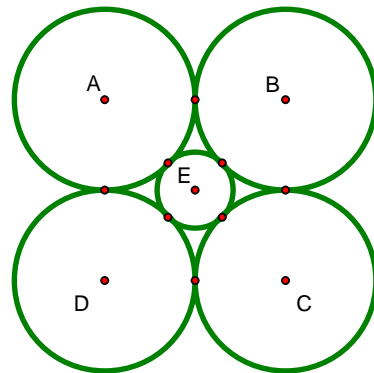
- A) 2    B) 3    C) 4    D) 5    E) 6

35) If hexagon ABCDEF has  $60^\circ$  degree rotational symmetry about its center P, then which one of the following is false?

- A) Quadrilateral ABCP is a rhombus  
 B) Quadrilateral ABCF is an isosceles trapezoid  
 C) Triangle ACE has rotational symmetry of  $60^\circ$   
 D) The reflection of ABCP about line  $\overline{AD}$  gives AFEP  
 E) Triangle ABP translated by vector  $\overrightarrow{BC}$  yields  $\triangle PCD$ .

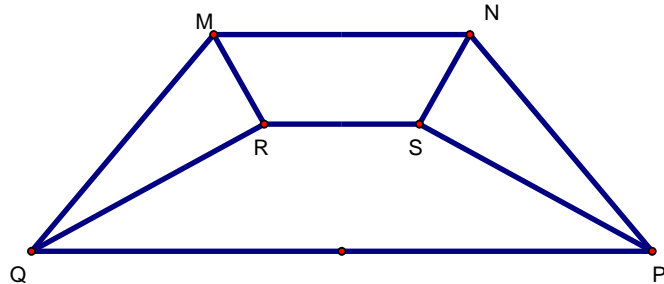
36) Circles with centers A, B, C, and D and common radius  $r$  are tangent to the circle with center E. Find the circumference of the circle with center E.

- A)  $(\sqrt{2} + 1)\pi r$   
 B)  $\pi r / (\sqrt{2} + 1)$   
 C)  $(2\sqrt{2} - 2)\pi r$   
 D)  $\pi r \sqrt{2}$   
 E)  $4\pi r$

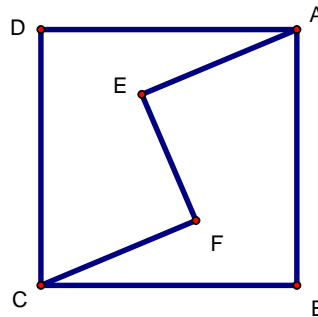




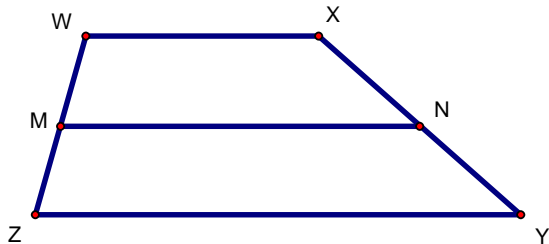
- 37) Trapezoids RSNM and MNPQ are similar with  $RS = 3$ ,  $m\angle MRQ = m\angle NSP = 90^\circ$ ,  $MQ = NP = 5$ . Find the number of square units in the area of quadrilateral RSNM.
- A) 24      B)  $\frac{68}{3}$       C)  $\frac{200}{3}$       D) 12      E) 14



- 38) In square ABCD,  $\overline{AE} \perp \overline{EF}$ ,  $\overline{EF} \perp \overline{FC}$ ,  $AE = 4$ ,  $EF = 9$ , and  $FC = 8$ . Find the perimeter of the square.
- A) 15      B)  $6\sqrt{2}$       C) 23      D)  $30\sqrt{2}$       E)  $21\sqrt{2}$



- 39) Given: WXYZ is a trapezoid with  $\overline{WX} \parallel \overline{ZY}$ ,  $\overline{MN}$  is the median,  $WX = 4x - 7$ ,  $MN = 2x + 10$ , and  $ZY = 2x + 1$ . Find the length of  $\overline{MN}$ .
- A) 13      B) 18      C) 27      D) 36      E) 45



- 40) In  $\triangle ABC$ ,  $\overrightarrow{CD}$  bisects  $\angle C$  so that  $AC = 6$ ,  $CB = 8$ , and  $DB = 4$ . Find the perimeter of  $\triangle ABC$ .
- A) 14      B) 17      C) 18      D) 21      E) 28

Extras. 42 and 43 have been used in the test.

- 42) D If the sides of a triangle are 20, 21, and 29 units, then the triangle is  
A) acute    B) isosceles    C) obtuse    D) right    E) not possible
- 43) E Given:  $\triangle ABC \sim \triangle BED$  with  $DB=4$ ,  $BE=6$ ,  $EC=2$ , and  $ED=4$ . Find the area of  $\triangle ABC$ .  
A) 14    B) 28    C) 56    D) 63    E)  $12\sqrt{7}$
- 44) E If ADEG is an isosceles trapezoid, BCEG is a rectangle, F is the midpoint of  $\overline{GE}$ ,  
 $AB=CD=9$ ,  $BF=CF=13$ , and  $GE=10$ , find the area of ADEG.  
A) 38    B) 44    C) 68    D) 120    E) 228