



## TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

### SIXTIETH ANNUAL MATHEMATICS CONTEST

2016

#### Geometry

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Scoring formula:  $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

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#### DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

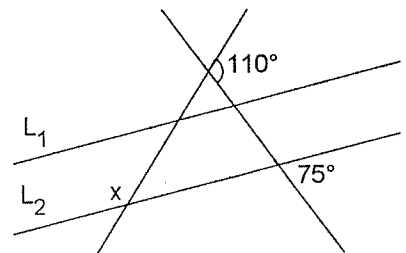
When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. The area of a circle is  $20\pi$ . Find the circumference of that circle.

- (a)  $400\pi$
- (b)  $100\pi$
- (c)  $10\pi$
- (d)  $4\sqrt{5}\pi$
- (e)  $2\sqrt{5}\pi$

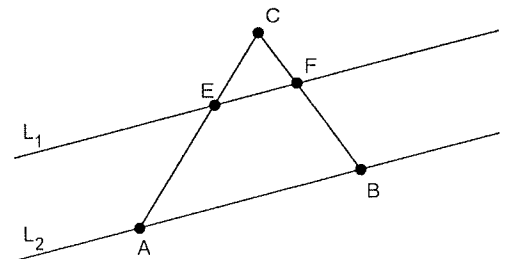
2. In the figure, lines  $L_1$  and  $L_2$  are parallel. Find the measure of angle  $x$ .

- (a)  $35^\circ$
- (b)  $70^\circ$
- (c)  $105^\circ$
- (d)  $135^\circ$
- (e)  $145^\circ$



3. Lines  $L_1$  and  $L_2$  are parallel.  $CF = 3$ ,  $CE = 4$ ,  $AC = 10$ , and  $AB = 8$ . Find the length  $EF$ .

- (a)  $\frac{16}{5}$
- (b)  $\frac{8}{5}$
- (c)  $\frac{5}{2}$
- (d) 4
- (e) 5



4. What is the equation for a circle centered at  $(-3,4)$  with diameter 14?

- (a)  $(x + 3)^2 + (y - 4)^2 = 49$
- (b)  $(x + 3)^2 + (y - 4)^2 = 7$
- (c)  $(x - 3)^2 + (y + 4)^2 = 49$
- (d)  $(x - 3)^2 + (y + 4)^2 = 7$
- (e)  $(x - 3)^2 + (y + 4)^2 = 196$

5. What is the measure of an exterior angle of a regular nonagon?

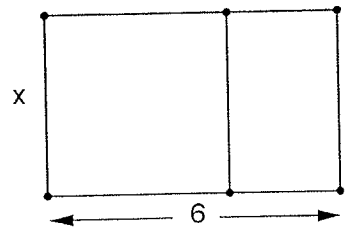
- (a)  $20^\circ$
- (b)  $30^\circ$
- (c)  $40^\circ$
- (d)  $140^\circ$
- (e)  $160^\circ$

6. A golden triangle is an isosceles triangle with the measure of either base angle equal to twice the measure of its vertex angle. What is the measure of the vertex angle in a golden triangle?

- (a)  $90^\circ$
- (b)  $72^\circ$
- (c)  $60^\circ$
- (d)  $45^\circ$
- (e)  $36^\circ$

7. The rectangle given is a golden rectangle. A golden rectangle is a rectangle such that when you cut a square off of one of the ends, you create a smaller rectangle that is similar to the original rectangle. Given the length of this golden rectangle is 6, find the width.

- (a)  $3 - 3\sqrt{5}$
- (b)  $3 + 3\sqrt{5}$
- (c)  $-3 - 3\sqrt{5}$
- (d)  $-3 + 3\sqrt{5}$
- (e)  $-3 + 6\sqrt{5}$

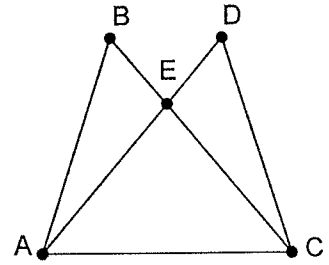


8. Two sides of a triangle have lengths 10 and 24. If the third side is also an integer, what is the maximum length it can be if the triangle is acute.

- (a) 33
- (b) 26
- (c) 25
- (d) 24
- (e) 21

9. Given that  $\overline{AB} \cong \overline{CD}$ , and  $\overline{BC} \cong \overline{AD}$ , what triangle congruence theorem could be used to prove that  $\angle B \cong \angle D$ ?

- (a) SSA
- (b) SAS
- (c) HL
- (d) SSS
- (e) It cannot be proven that  $\angle B \cong \angle D$



10. What is the contrapositive of the statement: "angles that are supplementary add to  $180^\circ$ "

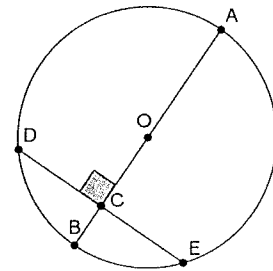
- (a) Angles that are not supplementary do not add to  $180^\circ$
- (b) Angles that add to  $180^\circ$  are supplementary
- (c) Angles that add to  $180^\circ$  are not supplementary
- (d) Angles that do not add to  $180^\circ$  are supplementary
- (e) Angles that do not add to  $180^\circ$  are not supplementary

11. Which of the following is never found outside of a triangle?

- (a) centroid
- (b) circumcenter
- (c) orthocenter
- (d) all of them
- (e) none of them

12. If  $BC = 6$  and  $DE = 24$ , find the length of the diameter  $AB$ .

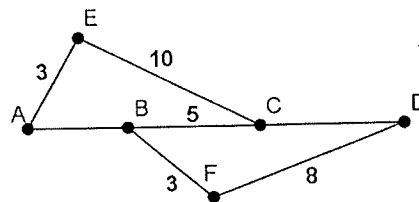
- (a) 15
- (b) 18
- (c) 27
- (d) 30
- (e) 36



13. If a dog is on a 20 foot leash, nailed to the corner of a rectangular building that is 15 feet long by 20 feet wide, what is the size of the area that the dog can reach?

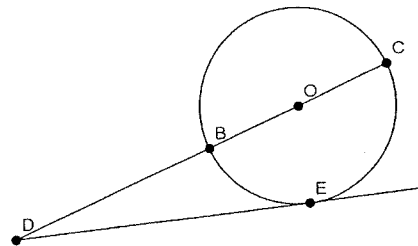
- (a)  $300\pi$  ft<sup>2</sup>
- (b)  $306.25\pi$  ft<sup>2</sup>
- (c)  $312.5\pi$  ft<sup>2</sup>
- (d)  $325\pi$  ft<sup>2</sup>
- (e)  $400\pi$  ft<sup>2</sup>

14. In the diagram (not drawn to scale), points A, B, C, and D are collinear, and points E and F lie on opposite sides of the line. The lengths AB and CD are both integers, where AB is made to be as small as possible and CD is made to be as large as possible. What is the length AD?



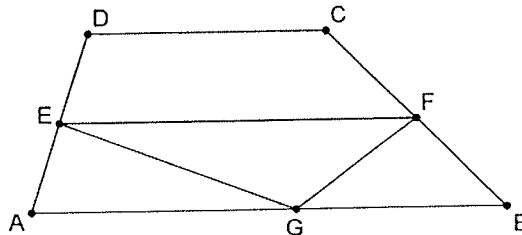
- (a) 14  
 (b) 13  
 (c) 12  
 (d) 10  
 (e) 8

15. In the figure, ray DE is tangent to the circle, segment DC goes through the center of the circle, and the measure of arc BE is  $40^\circ$ . What is the measure of angle D?



- (a)  $70^\circ$   
 (b)  $50^\circ$   
 (c)  $45^\circ$   
 (d)  $40^\circ$   
 (e)  $30^\circ$

16. The trapezoid ABCD has bases with lengths 12 and 9 and it has an area of 38.5. If  $\overline{EF}$  is the midsegment of the trapezoid, find the area of triangle EFG.



- (a)  $\frac{77}{2}$   
 (b)  $\frac{77}{3}$   
 (c)  $\frac{77}{4}$   
 (d)  $\frac{77}{6}$   
 (e)  $\frac{77}{8}$

17. A rectangular prism is made of glass and partially filled with colored water. Its dimensions are  $7\text{cm} \times 12\text{cm} \times 18\text{cm}$  and we will assume that the thickness of the walls is small enough to be ignored. When the box is placed on one of its  $7 \times 12$  sides, the water level is 15cm above the table. If the box is turned so that now it is lying on one of its  $7 \times 18$  sides, what will the water level above the table be?

- (a) 15 cm  
 (b) 14 cm  
 (c) 12 cm  
 (d) 10 cm  
 (e) 7 cm

18. Find the equation of the line that is the perpendicular bisector of the segment with endpoints  $(-2,8)$  and  $(6,2)$ .

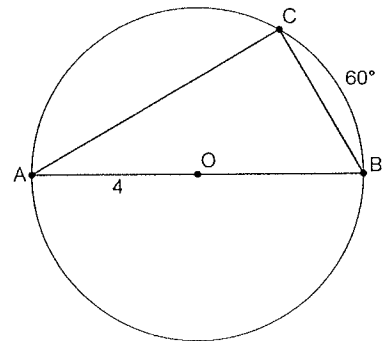
- (a)  $-4x + 3y = -14$
- (b)  $-4x + 3y = 7$
- (c)  $-4x + 3y = 13$
- (d)  $3x + 4y = 22$
- (e)  $3x + 4y = 26$

19. Mount Euclid is a perfect cone that is 10,000 feet tall. It erupts and blows off its top (also a cone), losing exactly  $\frac{1}{8}$  of its volume. What is the new height of Mount Euclid?

- (a) 5,000 feet
- (b) 6,125 feet
- (c) 7,500 feet
- (d) 8,250 feet
- (e) 8,750 feet

20. In the diagram, the circle has radius 4,  $\overline{AB}$  is a diameter, and arc  $BC$  has a measure of  $60^\circ$ . What is the area of  $\triangle ABC$ ?

- (a)  $2\sqrt{3}$
- (b)  $4\sqrt{3}$
- (c)  $8\sqrt{3}$
- (d)  $16\sqrt{3}$
- (e)  $32\sqrt{3}$



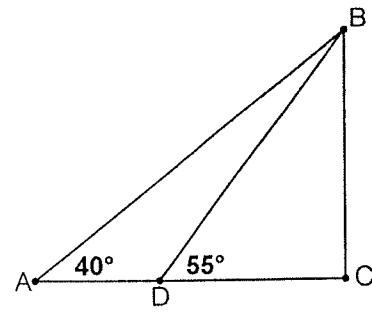
21. Let  $\triangle ABC$  be a right triangle, and let  $x$  be the measure of one of the acute angles. If

$\sin x = \frac{3}{4}$ , what is  $\cos x$ ?

- (a)  $\sqrt{7}$
- (b)  $\frac{\sqrt{7}}{4}$
- (c)  $\frac{4}{5}$
- (d)  $\frac{5}{4}$
- (e)  $\frac{4}{3}$

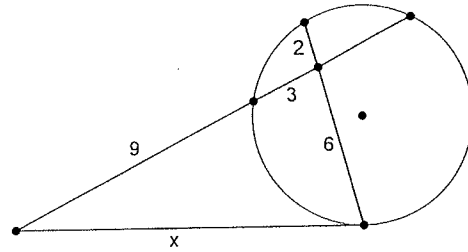
22. If angle C is a right angle and  $AC = 57$ , what is DC to the nearest integer?

- (a) 33
- (b) 34
- (c) 44
- (d) 45
- (e) 97



23. Given the figure on the right, what is  $x$  (assuming that the line containing the segment of length  $x$  is tangent to the circle)?

- (a) 4
- (b)  $3\sqrt{7}$
- (c) 12
- (d) 63
- (e) 144

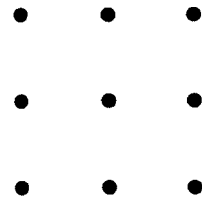


24. A rhombus has an area of 36, and one of its diagonals has a length of 12. Find the length of its sides.

- (a)  $3\sqrt{5}$
- (b) 6
- (c)  $6\sqrt{5}$
- (d)  $\frac{\sqrt{153}}{2}$
- (e)  $\sqrt{153}$

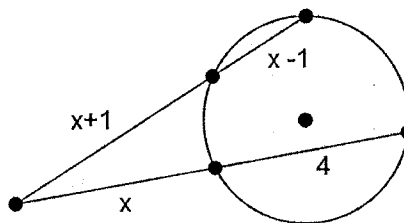
25. How many different isosceles triangles can be made if each vertex must be chosen from a point in the  $3 \times 3$  grid given?

- (a) 16
- (b) 20
- (c) 24
- (d) 28
- (e) 36



26. Given the diagram on the right, find  $x$ .

- (a) 1
- (b)  $\sqrt{2}$
- (c) 2
- (d)  $2 + \sqrt{5}$
- (e) 6



27. A girl is standing on a dock, holding a rope that is connected to an anchor on the bottom of a lake directly beneath her. She pulls the rope tight and it extends 1 meter above the dock. She is curious how far it is from the dock to the bottom of the lake, so she walks 7 meters down the dock until the rope, when held tightly, is even with the dock. Compute the distance from the dock to the bottom of the lake at her original position.

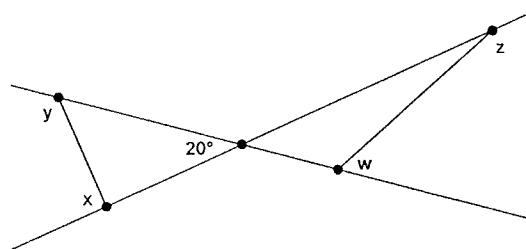
- (a) 22 meters
- (b) 23 meters
- (c) 24 meters
- (d) 25 meters
- (e) 26 meters

28. What is the distance between the line  $\frac{12}{5}x - 12 = y$  and the origin?

- (a) 5
- (b)  $\frac{65}{12}$
- (c)  $\frac{65}{14}$
- (d)  $\frac{60}{13}$
- (e) 13

29. Find the sum of the angles  $w$ ,  $x$ ,  $y$ , and  $z$ .

- (a) 320
- (b) 340
- (c) 360
- (d) 380
- (e) 400



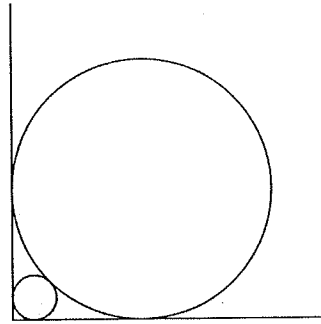


30. A square has vertices at  $(2,3)$ ,  $(5,7)$ ,  $(1,10)$ ,  $(-2,6)$ . If it is dilated with center of dilation at the origin, so that  $(2,3) \rightarrow (6,9)$ , what is the area of the new square?

- (a)  $\frac{25}{3}$
- (b) 25
- (c) 75
- (d) 144
- (e) 225

31. A circular table with radius 2 has been pushed into the corner of a room so that it is tangent to both walls. What would be the radius of a smaller circular table that is also tangent to the same two walls as well as the larger table?

- (a)  $\frac{1}{2}$
- (b)  $\sqrt{2} - 1$
- (c)  $2\sqrt{2} - 2$
- (d)  $6 - 4\sqrt{2}$
- (e)  $\frac{6-4\sqrt{2}}{3}$

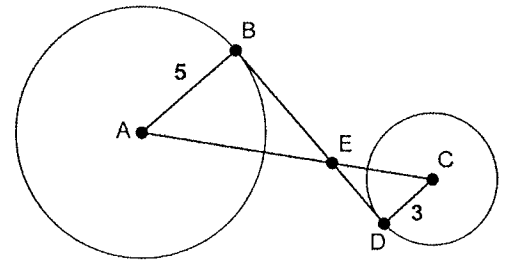


32. A sphere has radius  $r$ . Find the side length of a cube inscribed in the sphere in terms of  $r$ .

- (a)  $\frac{2r}{3}$
- (b)  $\frac{2\sqrt{3}r}{3}$
- (c)  $\frac{4r}{3}$
- (d)  $\frac{4r^2}{3}$
- (e)  $\frac{4\pi r}{3}$

33. Let  $\overline{BD}$  be the common internal tangent for the circles with centers A and C. If  $CE = 5$  find the length  $BD$ .

- (a)  $\frac{20}{3}$   
 (b)  $\frac{22}{3}$   
 (c)  $\frac{25}{3}$   
 (d)  $\frac{32}{3}$   
 (e)  $\frac{40}{3}$



34. A circle is inscribed in a quadrilateral (all four sides are tangent) that has side lengths of 5, 7, 10, and  $x$  meters. Which of the following could be the length  $x$ ?

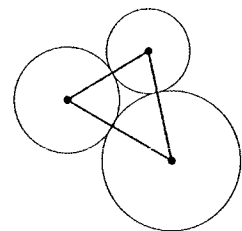
- (a) 5 meters  
 (b) 6 meters  
 (c) 7 meters  
 (d) 8 meters  
 (e) 9 meters

35. Point  $P = (3, 3\sqrt{3})$  is rotated counterclockwise about the origin by  $75^\circ$ . What are the coordinates after this rotation?

- (a)  $(3\sqrt{2}, 3\sqrt{2})$   
 (b)  $(-3\sqrt{2}, 3\sqrt{2})$   
 (c)  $(-3, 3\sqrt{3})$   
 (d)  $(3, 3\sqrt{3})$   
 (e)  $(-3\sqrt{3}, 3)$

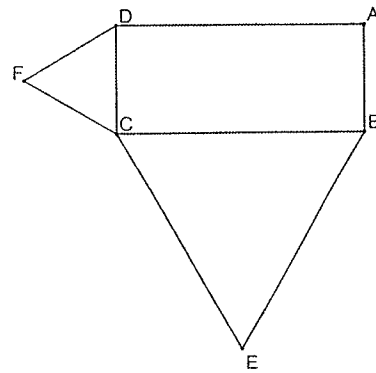
36. Three circles are cotangent with circumferences  $6\pi$ ,  $8\pi$ , and  $12\pi$ . Find the area of the triangle formed by the centers of the three circles. Round your answer to the nearest integer.

- (a) 31  
 (b) 45  
 (c) 52  
 (d) 62  
 (e) 90



37. Rectangle  $ABCD$  has side lengths 4 and  $\sqrt{3}$ . Equilateral triangles are constructed on adjacent sides and this creates triangle  $AEF$ . What is the area of this triangle?

- (a)  $\frac{15}{2}\sqrt{3}$   
 (b)  $\frac{31}{4}\sqrt{3}$   
 (c)  $8\sqrt{3}$   
 (d)  $\frac{33}{4}\sqrt{3}$   
 (e)  $\frac{17}{2}\sqrt{3}$

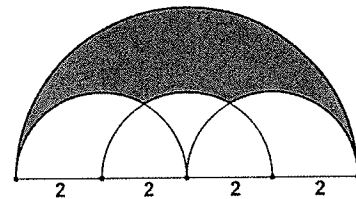


38. A rectangular pyramid has a volume of 720. If the length and the width of the base are 10 and 18, what is the surface area of the 4 sides combined?

- (a) 336  
 (b) 360  
 (c) 384  
 (d) 564  
 (e) 768

39. Find the shaded area in the figure, which is constructed of four semicircles.

- (a)  $\frac{14}{3}\pi - 2\sqrt{3}$   
 (b)  $\frac{14}{3}\pi - 4\sqrt{3}$   
 (c)  $\frac{14}{3}\pi + 4\sqrt{3}$   
 (d)  $\frac{20}{3}\pi - 2\sqrt{3}$   
 (e)  $\frac{20}{3}\pi - 4\sqrt{3}$



40. Two flag poles (one of height 6 meters and the other at height 18 meters) stick out of the ground at right angles. Ropes run from the base of each pole to the top of the other pole. How high above the ground will these two rope cross?

- (a) 2.5 meters  
 (b) 3 meters  
 (c) 3.5 meters  
 (d) 4 meters  
 (e) 4.5 meters

